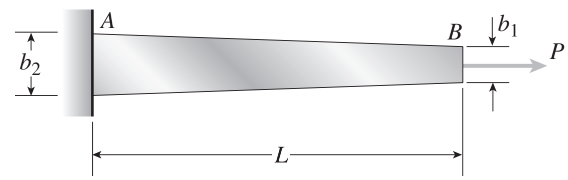
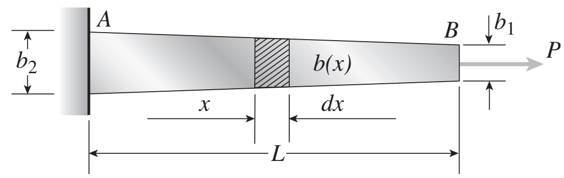
Problem 2.7-9 A slightly tapered bar AB of rectangular cross section and length L is acted upon by a force P (see figure). The width of the bar varies uniformly from b_2 at end A to b_1 at end B. The thickness t is constant.

(a) Determine the strain energy U of the bar.
(b) Determine the elongation δ of the bar by equating the strain energy to the work done by the force P.



Solution 2.7-9 **Tapered bar of rectangular cross section**



$$b(x) = b_2 - \frac{(b_2 - b_1)x}{L}$$

$$A(x) = tb(x)$$

$$= t\left[b_2 - \frac{(b_2 - b_1)x}{L}\right]$$

(a) Strain energy of the bar

$$U = \int \frac{[N(x)]^2 dx}{2EA(x)} \quad \text{(Eq. 2-41)}$$

$$= \int_0^L \frac{P^2 dx}{2Etb(x)} = \frac{P^2}{2Et}\int_0^L \frac{dx}{b_2 - (b_2 - b_1)\frac{x}{L}} \quad (1)$$

From Appendix C: $\displaystyle\int \frac{dx}{a + bx} = \frac{1}{b}\ln(a + bx)$

Apply this integration formula to Eq. (1):

$$U = \frac{P^2}{2Et}\left[\frac{1}{-(b_2 - b_1)(\frac{1}{L})}\ln\left[b_2 - \frac{(b_2 - b_1)x}{L}\right]\right]_0^L$$

$$= \frac{P^2}{2Et}\left[\frac{-L}{(b_2 - b_1)}\ln b_1 - \frac{-L}{(b_2 - b_1)}\ln b_2\right]$$

$$U = \frac{P^2 L}{2Et(b_2 - b_1)}\ln\frac{b_2}{b_1} \longleftarrow$$

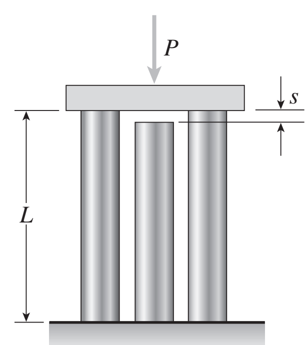
(b) Elongation of the bar (Eq. 2-42)

$$\delta = \frac{2U}{P} = \frac{PL}{Et(b_2 - b_1)}\ln\frac{b_2}{b_1} \longleftarrow$$

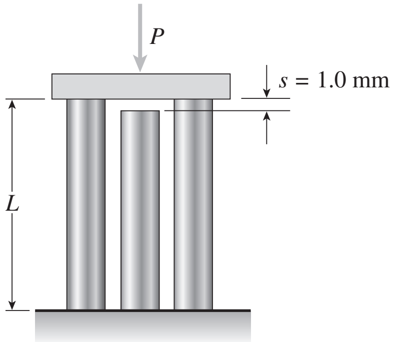
Note: This result agrees with the formula derived in prob. 2.3-11.

Problem 2.7-10 A compressive load P is transmitted through a rigid plate to three magnesium-alloy bars that are identical except that initially the middle bar is slightly shorter than the other bars (see figure). The dimensions and properties of the assembly are as follows: length $L = 1.0$ m, cross-sectional area of each bar $A = 3000$ mm^2, modulus of elasticity $E = 45$ GPa, and the gap $s = 1.0$ mm.

(a) Calculate the load P_1 required to close the gap.
(b) Calculate the downward displacement δ of the rigid plate when $P = 400$ kN.
(c) Calculate the total strain energy U of the three bars when $P = 400$ kN.
(d) Explain why the strain energy U is *not* equal to $P\delta/2$. (*Hint:* Draw a load-displacement diagram.)



Solution 2.7-10 Three bars in compression



$s = 1.0\,\text{mm}$

$L = 1.0\,\text{m}$

For each bar:

$A = 3000\,\text{mm}^2$

$E = 45\,\text{GPa}$

$\dfrac{EA}{L} = 135 \times 10^6\ \text{N/m}$

(a) LOAD P_1 REQUIRED TO CLOSE THE GAP

In general, $\delta = \dfrac{PL}{EA}$ and $P = \dfrac{EA\delta}{L}$

For two bars, we obtain:

$$P_1 = 2\left(\frac{EAs}{L}\right) = 2(135 \times 10^6\ \text{N/m})(1.0\ \text{mm})$$

$$P_1 = 270\ \text{kN} \longleftarrow$$

(b) DISPLACEMENT δ FOR $P = 400\,\text{kN}$

Since $P > P_1$, all three bars are compressed. The force P equals P_1 plus the additional force required to compress all three bars by the amount $\delta - s$.

$$P = P_1 + 3\left(\frac{EA}{L}\right)(\delta - s)$$

or $400\ \text{kN} = 270\ \text{kN} + 3(135 \times 10^6\ \text{N/m})(\delta - 0.001\,\text{m})$

Solving, we get $\delta = 1.321\ \text{mm} \longleftarrow$

(c) STRAIN ENERGY U FOR $P = 400\,\text{kN}$

$$U = \sum \frac{EA\delta^2}{2L}$$

Outer bars: $\delta = 1.321\,\text{mm}$

Middle bar: $\delta = 1.321\,\text{mm} - s$

$= 0.321\,\text{mm}$

$$U = \frac{EA}{2L}[2(1.321\ \text{mm})^2 + (0.321\ \text{mm})^2]$$

$$= \frac{1}{2}(135 \times 10^6\ \text{N/m})(3.593\ \text{mm}^2)$$

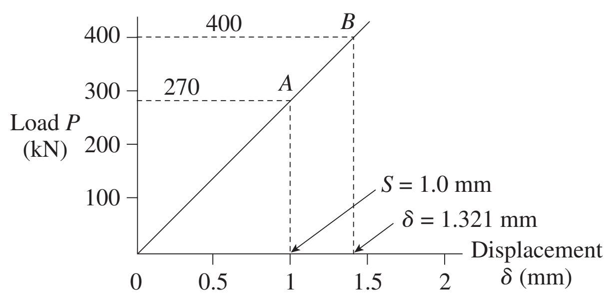
$$= 243\ \text{N} \cdot \text{m} = 243\ J \longleftarrow$$

(d) LOAD-DISPLACEMENT DIAGRAM

$U = 243\ \text{J} = 243\,\text{N} \cdot \text{m}$

$$\frac{P\delta}{2} = \frac{1}{2}(400\ \text{k}N)(1.321\ \text{mm}) = 264\ \text{N} \cdot \text{m}$$

The strain energy U is *not* equal to $\dfrac{P\delta}{2}$ because the load-displacement relation is not linear.

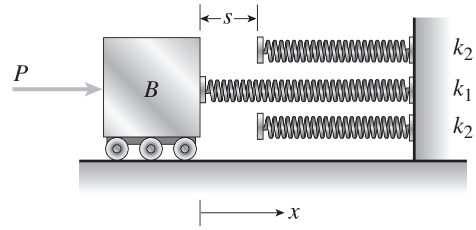


U = area under line OAB.

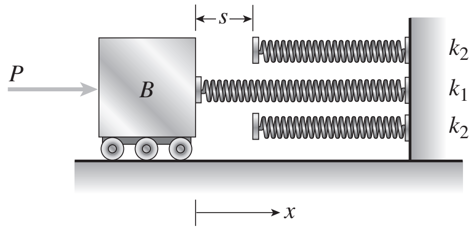
$\dfrac{P\delta}{2}$ = area under a straight line from O to B, which is larger than U.

Problem 2.7-11 A block B is pushed against three springs by a force P (see figure). The middle spring has stiffness k_1 and the outer springs each have stiffness k_2. Initially, the springs are unstressed and the middle spring
is longer than the outer springs (the difference in length is denoted s).

(a) Draw a force-displacement diagram with the force P as ordinate and the displacement x of the block as abscissa.
(b) From the diagram, determine the strain energy U_1 of the springs when $x = 2s$.
(c) Explain why the strain energy U_1 is not equal to $P\delta/2$, where $\delta = 2s$.



Solution 2.7-11 Block pushed against three springs



Force P_0 required to close the gap:

$$P_0 = k_1 s \tag{1}$$

FORCE-DISPLACEMENT RELATION BEFORE GAP IS CLOSED

$$P = k_1 x \qquad (0 \le x \le s)(0 \le P \le P_0) \tag{2}$$

FORCE-DISPLACEMENT RELATION AFTER GAP IS CLOSED

All three springs are compressed. Total stiffness equals $k_1 + 2k_2$. Additional displacement equals $x - s$. Force P equals P_0 plus the force required to compress all three springs by the amount $x - s$.

$$P = P_0 + (k_1 + 2k_2)(x - s)$$

$$= k_1 s + (k_1 + 2k_2)x - k_1 s - 2k_2 s$$

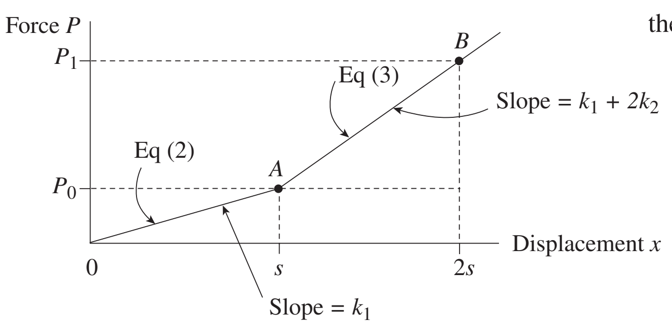
$$P = (k_1 + 2k_2)x - 2k_2 s \quad (x \ge s);\ (P \ge P_0) \tag{3}$$

P_1 = force P when $x = 2s$

Substitute $x = 2s$ into Eq. (3):

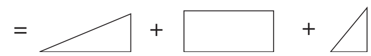
$$P_1 = 2(k_1 + k_2)s \tag{4}$$

(a) FORCE-DISPLACEMENT DIAGRAM



(b) STRAIN ENERGY U_1 WHEN $x = 2s$

U_1 = Area below force-displacement curve



$$= \frac{1}{2}P_0 s + P_0 s + \frac{1}{2}(P_1 - P_0)s = P_0 s + \frac{1}{2}P_1 s$$

$$= k_1 s^2 + (k_1 + k_2)s^2$$

$$U_1 = (2k_1 + k_2)s^2 \longleftarrow \tag{5}$$

(c) STRAIN ENERGY U_1 IS NOT EQUAL TO $\frac{P\delta}{2}$

For $\delta = 2s$: $\frac{P\delta}{2} = \frac{1}{2}P_1(2s) = P_1 s = 2(k_1 + k_2)s^2$

(This quantity is greater than U_1.)

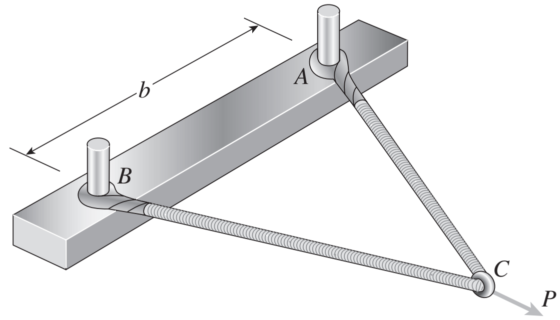
U = area under line OAB.

$\frac{P\delta}{2}$ = area under a straight line from O to B, which is larger U.

Thus, $\frac{P\delta}{2}$ is *not* equal to the strain energy because the force-displacement relation is not linear.

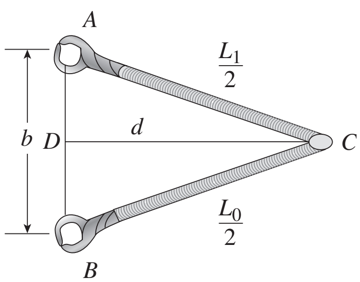
Problem 2.7-12 A bungee cord that behaves linearly elastically has an unstressed length $L_0 = 760$ mm and a stiffness $k = 140$ N/m. The cord is attached to two pegs, distance $b = 380$ mm apart, and pulled at its midpoint by a force $P = 80$ N (see figure).

(a) How much strain energy U is stored in the cord?
(b) What is the displacement δ_C of the point where the load is applied?
(c) Compare the strain energy U with the quantity $P\delta_C/2$.
(*Note:* The elongation of the cord is *not* small compared to its original length.)



Solution 2.7-12 Bungee cord subjected to a load P.

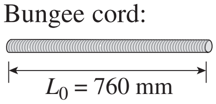
DIMENSIONS BEFORE THE LOAD P IS APPLIED



$L_0 = 760$ mm $\quad \frac{L_0}{2} = 380$ mm

$b = 380$ mm

Bungee cord:

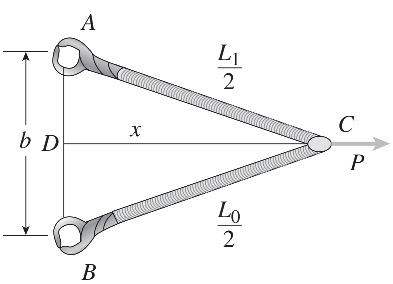


$k = 140$ N/m

From triangle *ACD:*

$$d = \frac{1}{2}\sqrt{L_0^2 - b^2} = 329.09 \text{ mm} \qquad (1)$$

DIMENSIONS AFTER THE LOAD P IS APPLIED



Let x = distance CD

Let L_1 = stretched length of bungee cord

From triangle *ACD*:

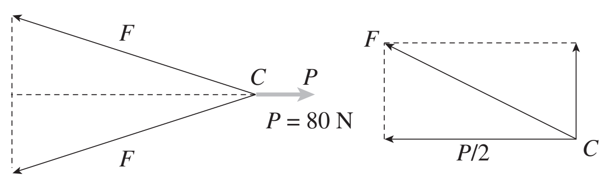
$P = 80$ N

$$\frac{L_1}{2} = \sqrt{\left(\frac{b}{2}\right)^2 + x^2} \qquad (2)$$

$$L_1 = \sqrt{b^2 + 4x^2} \qquad (3)$$

EQUILIBRIUM AT POINT C

Let F = tensile force in bungee cord



$$\frac{F}{P/2} = \frac{L_1/2}{x} \qquad F = \left(\frac{P}{2}\right)\left(\frac{L_1}{2}\right)\left(\frac{1}{x}\right)$$

$$= \frac{P}{2}\sqrt{1 + \left(\frac{b}{2x}\right)^2} \qquad (4)$$

ELONGATION OF BUNGEE CORD

Let δ = elongation of the entire bungee cord

$$\delta = \frac{F}{k} = \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} \qquad (5)$$

Final length of bungee cord = original length + δ

$$L_1 = L_0 + \delta = L_0 + \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} \qquad (6)$$

SOLUTION OF EQUATIONS

Combine Eqs. (6) and (3):

$$L_1 = L_0 + \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} = \sqrt{b^2 + 4x^2}$$

$$\text{or} \quad L_1 = L_0 + \frac{P}{4kx}\sqrt{b^2 + 4x^2} = \sqrt{b^2 + 4x^2}$$

$$L_0 = \left(1 - \frac{P}{4kx}\right)\sqrt{b^2 + 4x^2} \qquad (7)$$

This equation can be solved for x.

SUBSTITUTE NUMERICAL VALUES INTO EQ. (7):

$$760 \text{ mm} = \left[1 - \frac{(80 \text{ N})(1000 \text{ mm/m})}{4(140 \text{ N/m})x}\right] \times \sqrt{(380 \text{ mm})^2 + 4x^2} \qquad (8)$$

$$760 = \left(1 - \frac{142.857}{x}\right)\sqrt{144{,}400 + 4x^2} \quad (9) \qquad (9)$$

Units: x is in millimeters

Solve for x (Use trial & error or a computer program):

$x = 497.88$ mm

(a) STRAIN ENERGY U OF THE BUNGEE CORD

$$U = \frac{k\delta^2}{2} \qquad k = 140 \text{ N/m} \qquad P = 80 \text{ N}$$

From Eq. (5):

$$\delta = \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} = 305.81 \text{ mm}$$

$$U = \frac{1}{2}(140 \text{ N/m})(305.81 \text{ mm})^2 = 6.55 \text{ N} \cdot \text{m}$$

$$U = 6.55 \text{ J} \quad \longleftarrow$$

(b) DISPLACEMENT δ_C OF POINT C

$$\delta_C = x - d = 497.88 \text{ mm} - 329.09 \text{ mm}$$
$$= 168.8 \text{ mm} \quad \longleftarrow$$

(c) COMPARISON OF STRAIN ENERGY U WITH THE QUANTITY $P\delta_C/2$

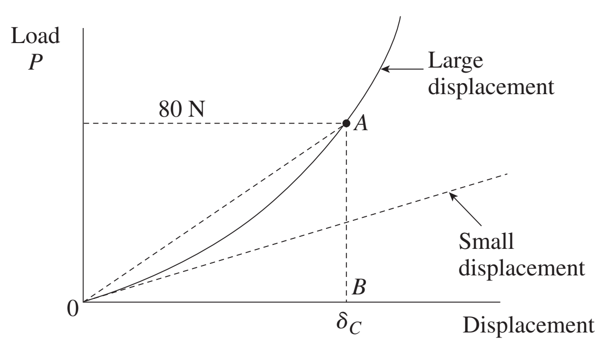
$U = 6.55$ J

$$\frac{P\delta_C}{2} = \frac{1}{2}(80 \text{ N})(168.8 \text{ mm}) = 6.75 \text{ J}$$

The two quantities are not the same. The work done by the load P is *not* equal to $P\delta_C/2$ because the load-displacement relation (see below) is non-linear when the displacements are large. (The *work* done by the load P *is* equal to the strain energy because the bungee cord behaves elastically and there are no energy losses.)

U = area OAB under the curve OA.

$\frac{P\delta_C}{2}$ = area of triangle OAB, which is greater than U.

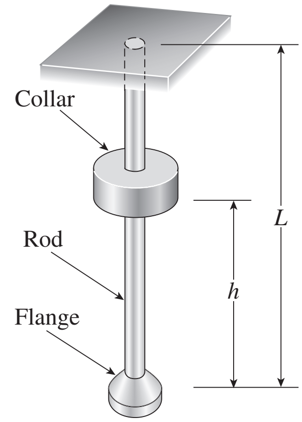


Impact Loading

The problems for Section 2.8 are to be solved on the basis of the assumptions and idealizations described in the text. In particular, assume that the material behaves linearly elastically and no energy is lost during the impact.

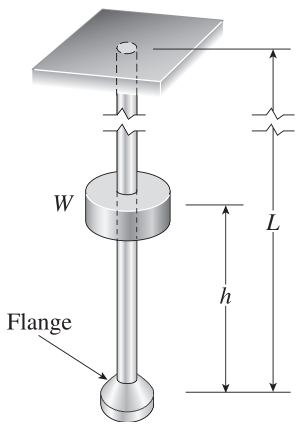
Problem 2.8-1 A sliding collar of weight $W = 150$ lb falls from a height $h = 2.0$ in. onto a flange at the bottom of a slender vertical rod (see figure). The rod has length $L = 4.0$ ft, cross-sectional area $A = 0.75$ in.^{2}, and modulus of elasticity $E = 30 \times 10^6$ psi.

Calculate the following quantities: (a) the maximum downward displacement of the flange, (b) the maximum tensile stress in the rod, and (c) the impact factor.



Probs. 2.8-1 and 2.8-3

Solution 2.8-1 Collar falling onto a flange



$W = 150$ lb

$h = 2.0$ in. $\qquad L = 4.0$ ft $= 48$ in.

$E = 30 \times 10^6$ psi $\qquad A = 0.75$ in.^{2}

(a) Downward displacement of flange

$$\delta_{st} = \frac{WL}{EA} = 0.00032 \text{ in.}$$

Eq. of (2-53):

$$\delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$$

$$= 0.0361 \text{ in.} \longleftarrow$$

(b) Maximum tensile stress (Eq. 2-55)

$$\sigma_{max} = \frac{E\delta_{max}}{L} = 22{,}600 \text{ psi} \longleftarrow$$

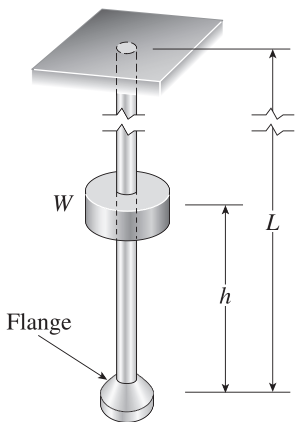
(c) Impact factor (Eq. 2-61)

$$\text{Impact factor} = \frac{\delta_{max}}{\delta_{st}} = \frac{0.0361 \text{ in.}}{0.00032 \text{ in.}}$$

$$= 113 \longleftarrow$$

Problem 2.8-2 Solve the preceding problem if the collar has mass $M = 80$ kg, the height $h = 0.5$ m, the length $L = 3.0$ m, the cross-sectional area $A = 350\ \text{mm}^2$, and the modulus of elasticity $E = 170$ GPa.

Solution 2.8-2 Collar falling onto a flange



$M = 80\,\text{kg}$

$W = Mg = (80\,\text{kg})(9.81\,\text{m/s}^2)$

$= 784.8\,\text{N}$

$h = 0.5\,\text{m}$ $L = 3.0\,\text{m}$

$E = 170\,\text{GPa}$ $A = 350\,\text{mm}^2$

(a) DOWNWARD DISPLACEMENT OF FLANGE

$$\delta_{st} = \frac{WL}{EA} = 0.03957\ \text{mm}$$

Eq. (2-53): $$\delta_{\max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$$

$$= 6.33\ \text{mm} \longleftarrow$$

(b) MAXIMUM TENSILE STRESS (EQ. 2-55)

$$\sigma_{\max} = \frac{E\delta_{\max}}{L} = 359\ \text{MPa} \longleftarrow$$

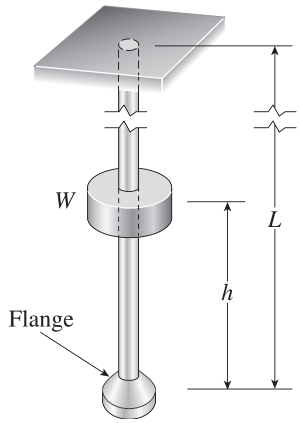
(c) IMPACT FACTOR (EQ. 2-61)

$$\text{Impact factor} = \frac{\delta_{\max}}{\delta_{st}} = \frac{6.33\ \text{mm}}{0.03957\ \text{mm}}$$

$$= 160 \longleftarrow$$

Problem 2.8-3 Solve Problem 2.8-1 if the collar has weight $W = 50$ lb, the height $h = 2.0$ in., the length $L = 3.0$ ft, the cross-sectional area $A = 0.25\ \text{in.}^2$, and the modulus of elasticity $E = 30{,}000$ ksi.

Solution 2.8-3 Collar falling onto a flange



$W = 50\,\text{lb}$ $h = 2.0\,\text{in.}$

$L = 3.0\,\text{ft} = 36\,\text{in.}$

$E = 30{,}000\,\text{psi}$ $A = 0.25\,\text{in.}^2$

(a) DOWNWARD DISPLACEMENT OF FLANGE

$$\delta_{st} = \frac{WL}{EA} = 0.00024\ \text{in.}$$

Eq. (2-53): $$\delta_{\max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$$

$$= 0.0312\ \text{in.} \longleftarrow$$

(b) MAXIMUM TENSILE STRESS (EQ. 2-55)

$$\sigma_{\max} = \frac{E\delta_{\max}}{L} = 26{,}000\ \text{psi} \longleftarrow$$

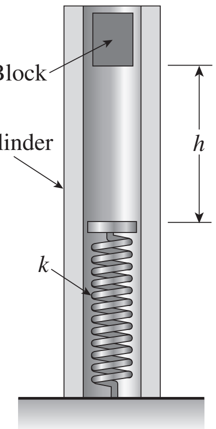
(c) IMPACT FACTOR (EQ. 2-61)

$$\text{Impact factor} = \frac{\delta_{\max}}{\delta_{st}} = \frac{0.0312\ \text{in.}}{0.00024\ \text{in.}}$$

$$= 130 \longleftarrow$$

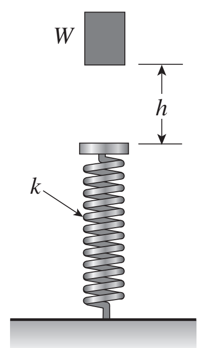
Problem 2.8-4 A block weighing $W = 5.0$ N drops inside a cylinder from a height $h = 200$ mm onto a spring having stiffness $k = 90$ N/m (see figure).

(a) Determine the maximum shortening of the spring due to the impact, and (b) determine the impact factor.



Prob. 2.8-4 and 2.8-5

Solution 2.8-4 Block dropping onto a spring



$W = 5.0\,\text{N} \qquad h = 200\,\text{mm} \qquad k = 90\,\text{N/m}$

(a) Maximum shortening of the spring

$$\delta_{st} = \frac{W}{k} = \frac{5.0\text{ N}}{90\text{ N/m}} = 55.56\text{ mm}$$

Eq. (2-53): $$\delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$$

$$= 215\text{ mm} \longleftarrow$$

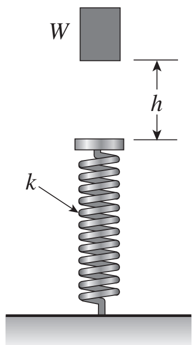
(b) Impact factor (Eq. 2-61)

$$\text{Impact factor} = \frac{\delta_{max}}{\delta_{st}} = \frac{215\text{ mm}}{55.56\text{ mm}}$$

$$= 3.9 \longleftarrow$$

Problem 2.8-5 Solve the preceding problem if the block weighs $W = 1.0$ lb, $h = 12$ in., and $k = 0.5$ lb/in.

Solution 2.8-5 Block dropping onto a spring



$W = 1.0\,\text{lb} \qquad h = 12\,\text{in.} \qquad k = 0.5\,\text{lb/in.}$

(a) MAXIMUM SHORTENING OF THE SPRING

$$\delta_{st} = \frac{W}{k} = \frac{1.0\text{ lb}}{0.5\text{ lb/in.}} = 2.0\text{ in.}$$

Eq. (2-53): $$\delta_{\max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$$

$$= 9.21\text{ in.} \longleftarrow$$

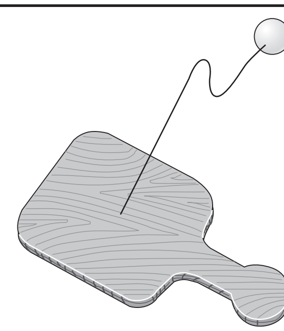
(b) IMPACT FACTOR (EQ. 2-61)

$$\text{Impact factor} = \frac{\delta_{\max}}{\delta_{st}} = \frac{9.21\text{ in.}}{2.0\text{ in.}}$$

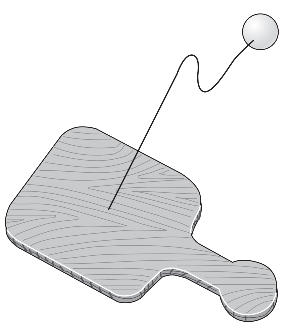
$$= 4.6 \longleftarrow$$

Problem 2.8-6 A small rubber ball (weight $W = 450$ mN) is attached by a rubber cord to a wood paddle (see figure). The natural length of the cord is $L_0 = 200$ mm, its cross-sectional area is $A = 1.6\text{ mm}^2$, and its modulus of elasticity is $E = 2.0$ MPa. After being struck by the paddle, the ball stretches the cord to a total length $L_1 = 900$ mm.

What was the velocity v of the ball when it left the paddle? (Assume linearly elastic behavior of the rubber cord, and disregard the potential energy due to any change in elevation of the ball.)



Solution 2.8-6 Rubber ball attached to a paddle



$g = 9.81\,\text{m/s}^2 \qquad E = 2.0\,\text{MPa}$

$A = 1.6\,\text{mm}^2 \qquad L_0 = 200\,\text{mm}$

$L_1 = 900\,\text{mm} \qquad W = 450\,\text{mN}$

WHEN THE BALL LEAVES THE PADDLE

$$KE = \frac{Wv^2}{2g}$$

WHEN THE RUBBER CORD IS FULLY STRETCHED:

$$U = \frac{EA\delta^2}{2L_0} = \frac{EA}{2L_0}(L_1 - L_0)^2$$

CONSERVATION OF ENERGY

$$KE = U \qquad \frac{Wv^2}{2g} = \frac{EA}{2L_0}(L_1 - L_0)^2$$

$$v^2 = \frac{gEA}{WL_0}(L_1 - L_0)^2$$

$$v = (L_1 - L_0)\sqrt{\frac{gEA}{WL_0}} \longleftarrow$$

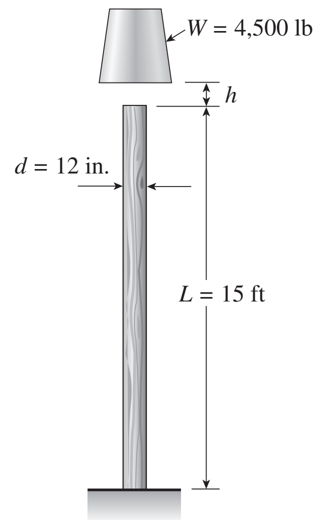
SUBSTITUTE NUMERICAL VALUES:

$$v = (700\text{ mm})\sqrt{\frac{(9.81\text{ m/s}^2)(2.0\text{ MPa})(1.6\text{ mm}^2)}{(450\text{ mN})(200\text{ mm})}}$$

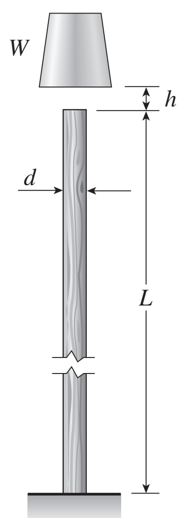
$$= 13.1\text{ m/s} \longleftarrow$$

Problem 2.8-7 A weight $W = 4500$ lb falls from a height h onto a vertical wood pole having length $L = 15$ ft, diameter $d = 12$ in., and modulus of elasticity $E = 1.6 \times 10^6$ psi (see figure).

If the allowable stress in the wood under an impact load is 2500 psi, what is the maximum permissible height h?



Solution 2.8-7 Weight falling on a wood pole



$W = 4500\,\text{lb} \qquad d = 12\,\text{in.}$

$L = 15\,\text{ft} = 180\,\text{in.}$

$$A = \frac{\pi d^2}{4} = 113.10\ \text{in.}^2$$

$E = 1.6 \times 10^6\,\text{psi}$

$\sigma_{allow} = 2500\,\text{psi}\ (= \sigma_{max})$

Find h_{max}

STATIC STRESS

$$\sigma_{st} = \frac{W}{A} = \frac{4500\ \text{lb}}{113.10\ \text{in.}^2} = 39.79\ \text{psi}$$

MAXIMUM HEIGHT h_{max}

Eq. (2–59): $$\sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$$

or

$$\frac{\sigma_{max}}{\sigma_{st}} - 1 = \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$$

Square both sides and solve for h:

$$h = h_{max} = \frac{L\sigma_{max}}{2E}\left(\frac{\sigma_{max}}{\sigma_{st}} - 2\right) \longleftarrow$$

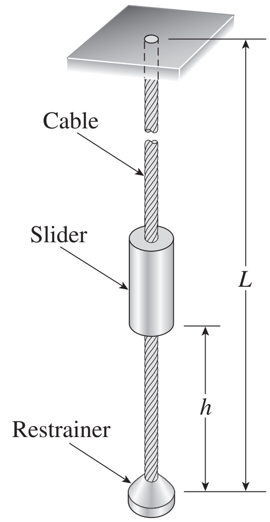
SUBSTITUTE NUMERICAL VALUES:

$$h_{max} = \frac{(180\ \text{in.})(2500\ \text{psi})}{2(1.6 \times 10^6\ \text{psi})}\left(\frac{2500\ \text{psi}}{39.79\ \text{psi}} - 2\right)$$

$$= 8.55\ \text{in.} \longleftarrow$$

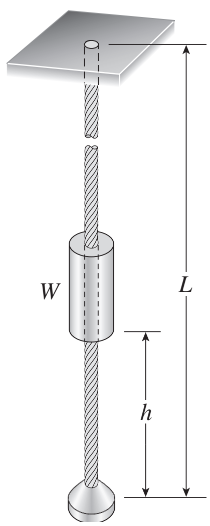
Problem 2.8-8 A cable with a restrainer at the bottom hangs vertically from its upper end (see figure). The cable has an effective cross-sectional area $A = 40\ \text{mm}^2$ and an effective modulus of elasticity $E = 130$ GPa. A slider of mass $M = 35$ kg drops from a height $h = 1.0$ m onto the restrainer.

If the allowable stress in the cable under an impact load is 500 MPa, what is the minimum permissible length L of the cable?



Probs. 2.8-8 and 2.8-9

Solution 2.8-8 Slider on a cable



$W = Mg = (35\ \text{kg})(9.81\ \text{m/s}^2) = 343.4\ \text{N}$

$A = 40\ \text{mm}^2 \qquad E = 130\ \text{GPa}$

$h = 1.0\ \text{m} \qquad \sigma_{allow} = \sigma_{max} = 500\ \text{MPa}$

Find minimum length L_{min}

STATIC STRESS

$$\sigma_{st} = \frac{W}{A} = \frac{343.4\ \text{N}}{40\ \text{mm}^2} = 8.585\ \text{MPa}$$

MINIMUM LENGTH L_{min}

Eq. (2–59): $$\sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$$

or

$$\frac{\sigma_{max}}{\sigma_{st}} - 1 = \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$$

Square both sides and solve for L:

$$L = L_{min} = \frac{2Eh\sigma_{st}}{\sigma_{max}(\sigma_{max} - 2\sigma_{st})} \longleftarrow$$

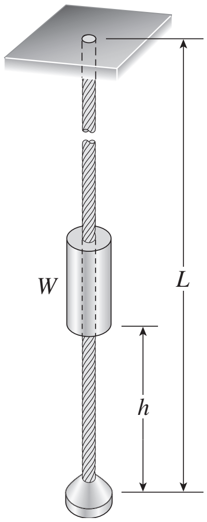
SUBSTITUTE NUMERICAL VALUES:

$$L_{min} = \frac{2(130\ \text{GPa})(1.0\ \text{m})(8.585\ \text{MPa})}{(500\ \text{MPa})[500\ \text{MPa} - 2(8.585\ \text{MPa})]}$$

$$= 9.25\ \text{mm} \longleftarrow$$

Problem 2.8-9 Solve the preceding problem if the slider has weight $W = 100$ lb, $h = 45$ in., $A = 0.080\ \text{in.}^2$, $E = 21 \times 10^6$ psi, and the allowable stress is 70 ksi.

Solution 2.8-9 **Slider on a cable**



$W = 100\,\text{lb}$

$A = 0.080\text{ in.}^2 \quad E = 21 \times 10^6\text{ psi}$

$h = 45\text{ in} \quad \sigma_{allow} = \sigma_{max} = 70\,\text{ksi}$

Find minimum length L_{min}

STATIC STRESS

$$\sigma_{st} = \frac{W}{A} = \frac{100\text{ lb}}{0.080\text{ in.}^2} = 1250\text{ psi}$$

MINIMUM LENGTH L_{min}

Eq. (2-59): $$\sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$$

or

$$\frac{\sigma_{max}}{\sigma_{st}} - 1 = \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$$

Square both sides and solve for L:

$$L = L_{min} = \frac{2Eh\sigma_{st}}{\sigma_{max}(\sigma_{max} - 2\sigma_{st})} \longleftarrow$$

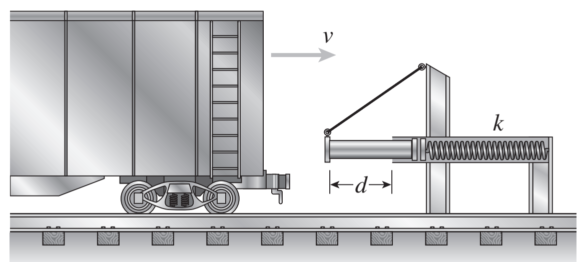
SUBSTITUTE NUMERICAL VALUES:

$$L_{min} = \frac{2(21 \times 10^6\text{ psi})(45\text{ in.})(1250\text{ psi})}{(70{,}000\text{ psi})[70{,}000\text{ psi} - 2(1250\text{ psi})]}$$

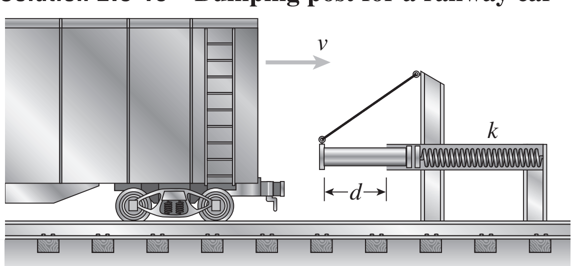
$$= 500\text{ in.} \longleftarrow$$

Problem 2.8-10 A bumping post at the end of a track in a railway yard has a spring constant $k = 8.0$ MN/m (see figure). The maximum possible displacement d of the end of the striking plate is 450 mm.

What is the maximum velocity v_{max} that a railway car of weight $W = 545$ kN can have without damaging the bumping post when it strikes it?



Solution 2.8-10 **Bumping post for a railway car**



$k = 8.0\,\text{MN/m} \qquad W = 545\,\text{kN}$

d = maximum displacement of spring

$d = \delta_{max} = 450\,\text{mm}$

Find v_{max}

KINETIC ENERGY BEFORE IMPACT

$$KE = \frac{Mv^2}{2} = \frac{Wv^2}{2g}$$

STRAIN ENERGY WHEN SPRING IS COMPRESSED TO THE MAXIMUM ALLOWABLE AMOUNT

$$U = \frac{k\delta_{max}^2}{2} = \frac{kd^2}{2}$$

CONSERVATION OF ENERGY

$$KE = U \qquad \frac{Wv^2}{2g} = \frac{kd^2}{2} \qquad v^2 = \frac{kd^2}{W/g}$$

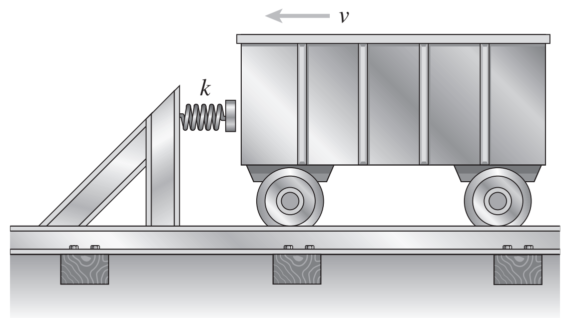
$$v = v_{max} = d\sqrt{\frac{k}{W/g}} \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

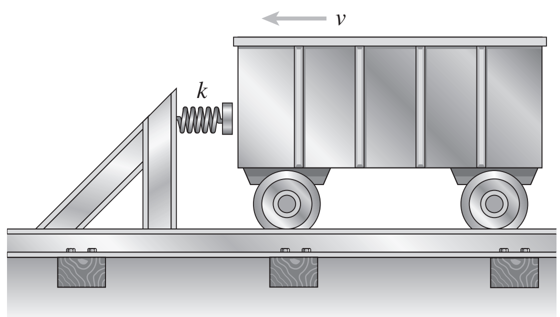
$$v_{max} = (450\text{ mm})\sqrt{\frac{8.0\text{ MN/m}}{(545\text{ kN})/(9.81\text{ m/s}^2)}}$$

$$= 5400\text{ mm/s} = 5.4\text{ m/s} \longleftarrow$$

Problem 2.8-11 A bumper for a mine car is constructed with a spring of stiffness $k = 1120$ lb/in. (see figure). If a car weighing 3450 lb is traveling at velocity $v = 7$ mph when it strikes the spring, what is the maximum shortening of the spring?



Solution 2.8-11 **Bumper for a mine car**



$k = 1120\,\text{lb/in.} \quad W = 3450\,\text{lb}$

$v = 7\,\text{mph} = 123.2\,\text{in./sec}$

$g = 32.2\,\text{ft/sec}^2 = 386.4\,\text{in./sec}^2$

Find the shortening δ_{max} of the spring.

KINETIC ENERGY JUST BEFORE IMPACT

$$KE = \frac{Mv^2}{2} = \frac{Wv^2}{2g}$$

STRAIN ENERGY WHEN SPRING IS FULLY COMPRESSED

$$U = \frac{k\delta_{max}^2}{2}$$

Conservation of energy

$$KE = U \qquad \frac{Wv^2}{2g} = \frac{k\delta_{max}^2}{2}$$

Solve for δ_{max}: $\quad \delta_{max} = \sqrt{\dfrac{Wv^2}{gk}} \longleftarrow$

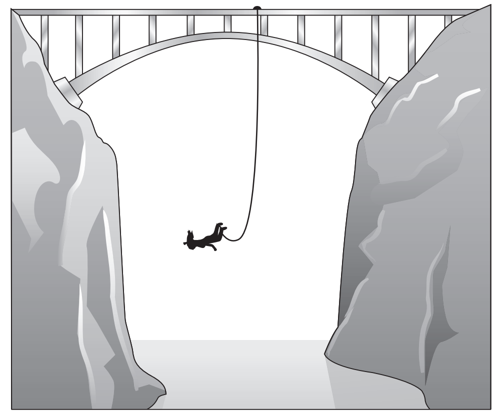
SUBSTITUTE NUMERICAL VALUES:

$$\delta_{max} = \sqrt{\frac{(3450\text{ lb})(123.2\text{ in./sec})^2}{(386.4\text{ in./sec}^2)(1120\text{ lb/in.})}}$$

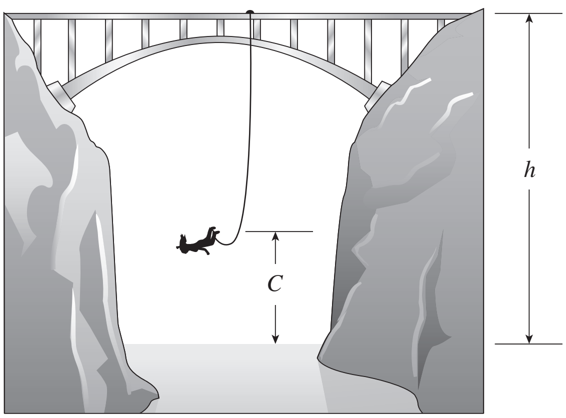
$$= 11.0\text{ in.} \longleftarrow$$

Problem 2.8-12 A bungee jumper having a mass of 55 kg leaps from a bridge, braking her fall with a long elastic shock cord having axial rigidity $EA = 2.3$ kN (see figure).

If the jumpoff point is 60 m above the water, and if it is desired to maintain a clearance of 10 m between the jumper and the water, what length L of cord should be used?



Solution 2.8-12 **Bungee jumper**



$W = Mg = (55\ \text{kg})(9.81\ \text{m/s}^2)$

$= 539.55\ \text{N}$

$EA = 2.3\ \text{kN}$

Height: $h = 60\ \text{m}$

Clearance: $C = 10\ \text{m}$

Find length L of the bungee cord.

$P.E.$ = Potential energy of the jumper at the top of bridge (with respect to lowest position)

$= W(L + \delta_{max})$

U = strain energy of cord at lowest position

$$= \frac{EA\delta_{max}^2}{2L}$$

CONSERVATION OF ENERGY

$$P.E. = U \quad W(L + \delta_{max}) = \frac{EA\delta_{max}^2}{2L}$$

$$\text{or} \quad \delta_{max}^2 - \frac{2WL}{EA}\delta_{max} - \frac{2WL^2}{EA} = 0$$

SOLVE QUADRATIC EQUATION FOR δ_{max}:

$$\delta_{max} = \frac{WL}{EA} + \left[\left(\frac{WL}{EA}\right)^2 + 2L\left(\frac{WL}{EA}\right)\right]^{1/2}$$

$$= \frac{WL}{EA}\left[1 + \left(1 + \frac{2EA}{W}\right)^{1/2}\right]$$

VERTICAL HEIGHT

$$h = C + L + \delta_{max}$$

$$h - C = L + \frac{WL}{EA}\left[1 + \left(1 + \frac{2EA}{W}\right)^{1/2}\right]$$

SOLVE FOR L:

$$L = \frac{h - C}{1 + \frac{W}{EA}\left[1 + \left(1 + \frac{2EA}{W}\right)^{1/2}\right]} \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\frac{W}{EA} = \frac{539.55\ \text{N}}{2.3\ \text{kN}} = 0.234587$$

Numerator $= h - c = 60\ \text{m} - 10\ \text{m} = 50\ \text{m}$

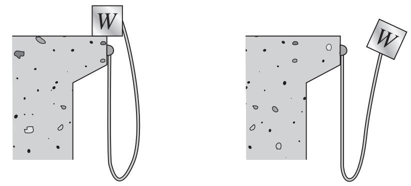
$$\text{Denominator} = 1 + (0.234587) \times \left[1 + \left(1 + \frac{2}{0.234587}\right)^{1/2}\right]$$

$$= 1.9586$$

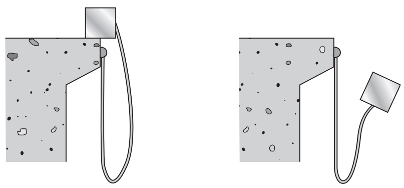
$$L = \frac{50\ \text{m}}{1.9586} = 25.5\ \text{m} \longleftarrow$$

Problem 2.8-13 A weight W rests on top of a wall and is attached to one end of a very flexible cord having cross-sectional area A and modulus of elasticity E (see figure). The other end of the cord is attached securely to the wall. The weight is then pushed off the wall and falls freely the full length of the cord.

(a) Derive a formula for the impact factor.
(b) Evaluate the impact factor if the weight, when hanging statically, elongates the band by 2.5% of its original length.



Solution 2.8-13 **Weight falling off a wall**



W = Weight

Properties of Elastic cord:

E = modulus of elasticity

A = cross-sectional area

L = original length

δ_{max} = elongation of elastic cord

$P.E.$ = potential energy of weight before fall (with respect to lowest position)

$P.E. = W(L + \delta_{max})$

Let U = strain energy of cord at lowest position

$$U = \frac{EA\delta_{max}^2}{2L}$$

CONSERVATION OF ENERGY

$$P.E. = U \qquad W(L + \delta_{max}) = \frac{EA\delta_{max}^2}{2\,L}$$

$$\text{or} \quad \delta_{max}^2 - \frac{2WL}{EA}\delta_{max} - \frac{2WL^2}{EA} = 0$$

SOLVE QUADRATIC EQUATION FOR δ_{max}:

$$\delta_{max} = \frac{WL}{EA} + \left[\left(\frac{WL}{EA}\right)^2 + 2L\left(\frac{WL}{EA}\right)\right]^{1/2}$$

STATIC ELONGATION

$$\delta_{st} = \frac{WL}{EA}$$

IMPACT FACTOR

$$\frac{\delta_{max}}{\delta_{st}} = 1 + \left[1 + \frac{2EA}{W}\right]^{1/2} \longleftarrow$$

NUMERICAL VALUES

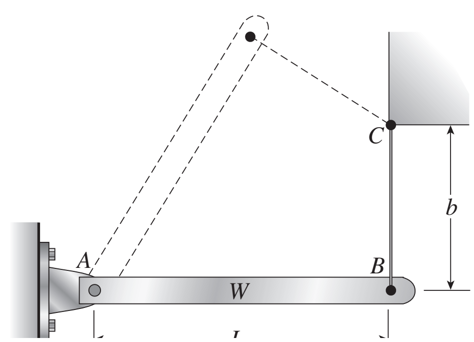
$\delta_{st} = (2.5\%)(L) = 0.025L$

$$\delta_{st} = \frac{WL}{EA} \qquad \frac{W}{EA} = 0.025 \qquad \frac{EA}{W} = 40$$

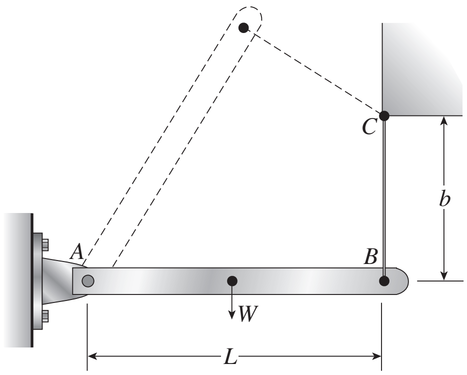
Impact factor $= 1 + [1 + 2(40)]^{1/2} = 10 \longleftarrow$

Problem 2.8-14 A rigid bar AB having mass $M = 1.0$ kg and length $L = 0.5$ m is hinged at end A and supported at end B by a nylon cord BC (see figure). The cord has cross-sectional area $A = 30\ \text{mm}^2$, length $b = 0.25$ m, and modulus of elasticity $E = 2.1$ GPa.

If the bar is raised to its maximum height and then released, what is the maximum stress in the cord?



Solution 2.8-14 Falling bar AB



RIGID BAR:

$W = Mg = (1.0\,\text{kg})(9.81\,\text{m/s}^2)$

$= 9.81$ N

$L = 0.5$ m

NYLON CORD:

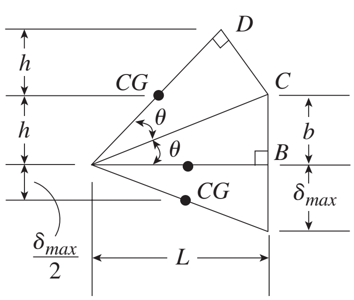
$A = 30\,\text{mm}^2$

$b = 0.25$ m

$E = 2.1$ GPa

Find maximum stress σ_{max} in cord BC.

GEOMETRY OF BAR AB AND CORD BC



$\overline{CD} = \overline{CB} = b$

$\overline{AD} = \overline{AB} = L$

h = height of center of gravity of raised bar AD

δ_{max} = elongation of cord

From triangle ABC: $\sin\theta = \dfrac{b}{\sqrt{b^2 + L^2}}$

$$\cos\theta = \frac{L}{\sqrt{b^2 + L^2}}$$

From line AD : $\sin 2\theta = \dfrac{2h}{AD} = \dfrac{2h}{L}$

From Appendix C: $\sin 2\theta = 2\sin\theta\cos\theta$

$$\therefore\ \frac{2h}{L} = 2\left(\frac{b}{\sqrt{b^2+L^2}}\right)\left(\frac{L}{\sqrt{b^2+L^2}}\right) = \frac{2bL}{b^2+L^2}$$

and $\quad h = \dfrac{bL^2}{b^2+L^2}$ (Eq. 1)

CONSERVATION OF ENERGY

$P.E.$ = potential energy of raised bar AD

$$= W\left(h + \frac{\delta_{max}}{2}\right)$$

U = strain energy of stretched cord $= \dfrac{EA\delta_{max}^2}{2b}$

$P.E. = U \quad W\left(h + \dfrac{\delta_{max}}{2}\right) = \dfrac{EA\delta_{max}^2}{2b}$ (Eq. 2)

For the cord: $\delta_{max} = \dfrac{\sigma_{max} b}{E}$

Substitute into Eq. (2) and rearrange:

$\sigma_{max}^2 - \dfrac{W}{A}\sigma_{max} - \dfrac{2WhE}{bA} = 0$ (Eq. 3)

Substitute from Eq. (1) into Eq. (3):

$\sigma_{max}^2 - \dfrac{W}{A}\sigma_{max} - \dfrac{2WL^2E}{A(b^2+L^2)} = 0$ (Eq. 4)

SOLVE FOR σ_{max}:

$$\sigma_{max} = \frac{W}{2A}\left[1 + \sqrt{1 + \frac{8L^2EA}{W(b^2+L^2)}}\right] \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$\sigma_{max} = 33.3$ MPa $\longleftarrow$